Homework for January 23, 2023

The homework is to be sent to frederic.vivien@inria.fr.

1 Classic pebble game

Consider a pyramid graph, obtained by slicing a 2D $n \times n$ mesh along its diagonal.



Questions:

- 1. Describe a pebbling strategy (without shift) that pebbles the *n*-level pyramid using only n + 1 pebbles (for $n \ge 2$).
- 2. Prove that any pebbling strategy (without shift) that pebbles this graph uses at least n+1 pebbles (for $n \ge 2$).

2 Lower bound on the volume of communications

We consider the following algorithm for computing the solution of a linear system of equations Ax = bwhere A is a *lower triangular* matrix (of size $n \times n$) and x and b are two vectors (of size n):

Questions:

- 1. Show how this computation can be modeled as a generalized matrix computation. In particular, exhibit $A, B, C, f_{i,j}, g_{i,j,k}, S_{i,j}$ and possibly other arguments.
- 2. Establish a lower bound on the total volume of communication for this computation.