## Homework for January 23, 2023

The homework is to be sent to frederic.vivien@inria.fr.

## 1 Classic pebble game

Consider a pyramid graph, obtained by slicing a $2 \mathrm{D} n \times n$ mesh along its diagonal.


## Questions:

1. Describe a pebbling strategy (without shift) that pebbles the $n$-level pyramid using only $n+1$ pebbles (for $n \geq 2$ ).
2. Prove that any pebbling strategy (without shift) that pebbles this graph uses at least $n+1$ pebbles (for $n \geq 2$ ).

## 2 Lower bound on the volume of communications

We consider the following algorithm for computing the solution of a linear system of equations $A x=b$ where $A$ is a lower triangular matrix (of size $n \times n$ ) and $x$ and $b$ are two vectors (of size $n$ ):

$$
\begin{aligned}
& \text { for } i=1 \ldots n \text { do } \\
& L x_{i} \leftarrow b_{i} \\
& \text { for } i=1 \ldots n \text { do } \\
& \qquad \begin{array}{l}
x_{i} \leftarrow \frac{x_{i}}{A_{i, i}} ; \\
\text { for } k=i+1 \ldots n \text { do } \\
\quad x_{k} \leftarrow x_{k}-x_{i} \times A_{k, i}
\end{array}
\end{aligned}
$$

## Questions:

1. Show how this computation can be modeled as a generalized matrix computation. In particular, exhibit $A, B, C, f_{i, j}, g_{i, j, k}, S_{i, j}$ and possibly other arguments.
2. Establish a lower bound on the total volume of communication for this computation.
